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The effect of the macroscopic local inertial term on the non-Newtonian fluid flow in channels filled with porous medium

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Abstract

The transient hydrodynamics behavior of the non-Newtonian fluid flow in horizontal parallel-plate channels filled with porous medium is investigated numerically. The role of the macroscopic local inertial term in the porous domain momentum equation is studied. It is found that the macroscopic local inertial term has insignificant effect on the channel hydrodynamics behavior for all non-Newtonian fluids having power law index less than 0.5 and over the entire range of Darcy and Forchheimer numbers. However, the macroscopic local inertial term has significant effect when the power law index is greater than 1 over a wide range of Darcy and Forchheimer numbers especially for relatively high values of Darcy and low values of Forchheimer numbers. It is found that the effect of the macroscopic local inertial term is very sensitive to the Forchheimer number at high values of Darcy numbers and power law index. Also, there is an upper limit for *n* beyond which changing the power law index has insignificant effect on the local inertial term.

Keywords: Porous media; Local inertial term; Transient flow; Non-Newtonian fluids; Macroscopic local inertial term

1. Introduction

The flow of non-Newtonian fluids through a porous medium is a topic of special interest in many engineering applications. Examples of these applications are filtration processes, biomechanics, packed bed reactors, geothermal engineering, insulation system, ceramic processing, enhanced oil recovery, chromatography and many others [1,2].

The literature shows that several investigators have studied the characteristics of the hydrodynamics as well as the thermal behavior of non-Newtonian flows through porous channels. Examples of these investigations may be found in [1–6].

In the present work, the transient hydrodynamics characteristics of a non-Newtonian fluid flow inside horizontal parallel-plate channels filled with porous medium is investigated numerically. The unsteadiness in the fluid flow is due to a suddenly imposed pressure gradient which drives the flow.

The main goal of the present study is to investigate the role of the macroscopic local inertial term in the porous domain momentum equation and its effect on the hydrodynamics behavior of a non-Newtonian fluid flow in porous channels. In the literature about fluid flow in porous domains, it has been realized that the macroscopic local inertial term is usually small compared to the microscopic Darcy drag term, and hence can be neglected [7–15]. In most practical situations in porous domain applications, the velocity responds to an imposed pressure change within a second or less. The macroscopic local inertial term may be important if an oscillatory pressure gradient is imposed or if the porous domain is of large void fraction [7,11,12,15]. The present investigations focus on the operating and geometrical parameters within which the macroscopic local inertial term may be significant. Such an investigation for a

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Nomenclature

C	coefficient in the modified dimensionless Forchheimer coefficient
Da^*	Darcy number, $K^*/\epsilon^n L^{n+1}$
K	intrinsic permeability of the porous medium
K^*	modified permeability of the porous medium for flow of power law fluids
$2L$	channel width
n	power law index
p	pressure
t	time
t_0	reference time, $\rho\epsilon^{n-1}L^{n+1}/\mu^* u_0^{n-1}$
t_t	transient time
u	axial velocity
u_0	reference axial velocity, $\sqrt[n]{(-\mathrm{d}p/\mathrm{d}x)\epsilon^n(L^{n+1}/\mu^*)}$
U	dimensionless axial velocity, u/u_0
x	axial coordinate
y	transverse coordinate
Y	dimensionless transverse coordinate, y/L
Greek symbols	
Γ	dimensionless Forchheimer coefficient, $\rho C\epsilon^n L^{n+1}u_0^{2-n}/\mu^*\sqrt{K}$
ϵ	porosity of the porous domain
μ	dynamic viscosity
μ^*	consistency index of a power law fluid
ν	kinematics viscosity
ρ	fluid density
τ	dimensionless time, t/t_0
τ_t	dimensionless transient time, t_t/t_0

non-Newtonian fluid flow in porous domain is not available yet in the literature. Also, the effect of these geometrical and operating conditions on the transient time, which is defined as the time required by the flow to attain steady state behavior, need to be investigated.

In this study, the Darcy–Brinkman–Forchheimer model is adopted to describe the non-Newtonian fluid flow hydrodynamic behavior. The inclusion of the Brinkman term is justified when the porous domain has large void fraction, i.e., $\epsilon > 0.6$ [14]. The non-Newtonian fluid is assumed to obey the power law constitution.

2. Mathematical formulation

Consider an unsteady laminar forced non-Newtonian fluid flow into a horizontal parallel-plate channel filled with porous medium. The unsteadiness in the fluid flow is due to a suddenly imposed pressure gradient which drives the flow. The fluid is assumed to be non-Newtonian which obeys the power law with uniform properties and the porous medium is isotropic and homogeneous. The flow is assumed to be hydrodynamically fully developed which velocity does not depend on the axial direction x of the channel. As a result of the continuity equation, the flow is a unidirectional one and it is expressed in terms of the axial velocity u alone. Also, the pressure gradient is assumed to be constant. Using the dimensionless parameters given in the nomenclature, the equation of motion is given as [2,4,15]:

$$\frac{\partial U}{\partial \tau} = 1 + \frac{\partial}{\partial Y}\left[\left|\frac{\partial U}{\partial Y}\right|^{n-1}\frac{\partial U}{\partial Y}\right] - \frac{1}{Da^*}U^n - \Gamma U^2 \tag{1}$$

where

$$Da^* = \frac{K^*}{\epsilon^n L^{n+1}}, \quad \Gamma = \frac{C\epsilon^n L^{n+1}u_0^{2-n}\rho}{\mu^*\sqrt{K}}$$

Eq. (1) has the following initial and boundary conditions:

$$U(0, Y) = 0 \tag{2}$$

$$U(\tau, -1) = U(\tau, 1) = 0 \tag{3}$$

In Eq. (1), the acceleration coefficient tensor is assumed to be $1/\epsilon^n$ [2].

3. Numerical method

The one-dimensional unsteady governing equation is solved numerically using the finite-volume approach [16]. To achieve diagonal dominance, the source term S_u is linearized such as its S_p coefficient is negative. The source term S_u represents the Darcy term, Forchheimer term and the dimensionless pressure gradient ($=1$), and it is given as:

$$S_u = S_p U + S_c = 1 - \frac{1}{Da^*}U^n - \Gamma U^2$$

As a result, the coefficients S_p and S_c are given as:

$$S_p = -\frac{1}{Da^*}U^{n-1} - \Gamma U, \quad S_c = 1$$

Such representation satisfies the boundedness condition, which considered a sufficient condition for a convergence. The central differencing of the diffusion term, which refers to the Brinkman term, assures the conservativeness property of the scheme. Eq. (1) is discretized with an implicit Euler time scheme. The resulting system of algebraic equations must be solved at each time level. The time marching procedure starts with a given initial field of velocity. The accuracy of the scheme is only first-order in time, thus small dimensionless time steps are needed to ensure the accuracy of results. The implicit scheme is considered robust and unconditionally stable. The above discretization procedure led to a tri-diagonal system. Thomas tri-diagonal matrix algorithm (TDMA) is used for solution. This method considered inexpensive and requires a minimum amount of storage.

A uniform grid is used for the dimensionless spatial coordinate; the adequacy of the grid is verified by comparing the results of different choices, 50 grids (nodes) are found to be satisfactory. The dimensionless time step is used based on number of refinements to ensure that the results are independent of such choices. The tests are performed for several combinations of Da^*, Γ and n. Values of dimensionless time steps from 10^{-3} to 10^{-5} are considered. Dimensionless time steps of 10^{-4} are found adequate for this study. The numerical iteration is repeated until the rate of change of the maximum velocity reaches a tolerance of 10^{-4}.

4. Results and discussion

To verify the validity of the adopted numerical scheme, the steady state version of Eqs. (1)–(3), for Newtonian fluid ($n = 1$) and with negligible microscopic inertial term ($\Gamma = 0$), is solved analytically. The analytical solution under these assumptions is given as:

$$U(Y) = Da^* \left[1 - \frac{\cosh\left(\frac{Y}{\sqrt{Da^*}}\right)}{\cosh\left(\frac{1}{\sqrt{Da^*}}\right)} \right] \tag{4}$$

Fig. 1 shows a comparison between the steady state numerical and analytical velocity profiles for $\Gamma = 0$ and $n = 1$. As clear from this figure, the results are in excellent agreement. Another verification is made by obtaining the analytical transient solution for Eqs. (1)–(3) with $n = 1$, $\Gamma = 0$ and $Da^* \to \infty$. This analytical solution is given as:

$$U(\tau, Y) = \frac{1}{2}(1 - Y^2) - \sum_{n=1}^{\infty} \frac{2\sin(\beta_n)}{\beta_n^3} e^{-\beta_n^2 \tau} \cos(\beta_n Y) \tag{5}$$

where

$$\beta_n = (2n - 1)\frac{\pi}{2}$$

Also, the numerical results are found to be in excellent agreement with the analytical ones as shown in Fig. 2.

In the following figures, the focus is on the effect of different parameters, such as Da^*, Γ and n, on the transient time τ_t. The transient time is defined as the time required by the channel to attain approximately the steady state velocity distribution. It is clear that this time depends on the location Y within the channel. However, it is noticed that the channel center, which has the maximum velocity, also has the longest transient time τ_t. Also, it is noticed that the transient time is more

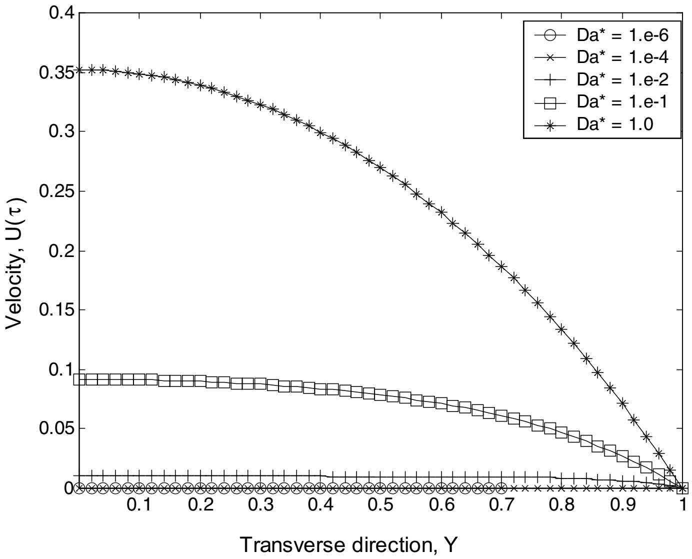

Fig. 1. Comparison between the steady state numerical (symbols) and analytical (lines) velocity profiles at different Da^*, for $\Gamma = 0$ and $n = 1$.

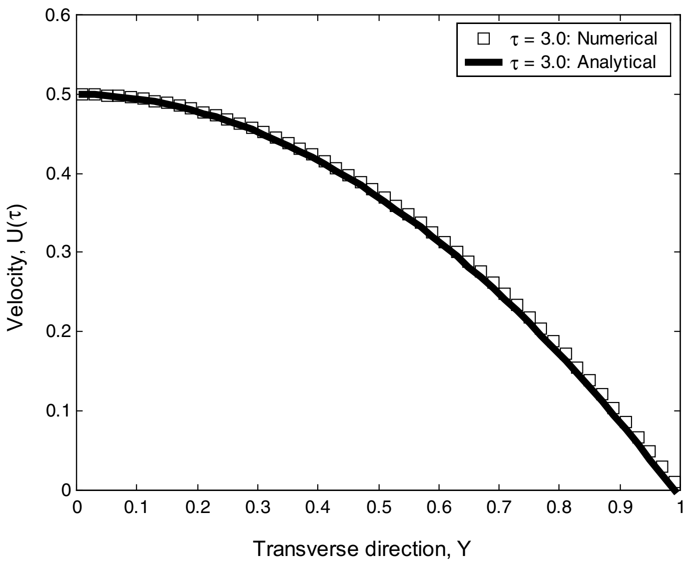

Fig. 2. Comparison between the transient numerical and analytical velocity profiles at $\tau = 3$, $n = 1$, $\Gamma = 0$ and $Da^* = \infty$.

sensitive to the effect of other parameters at this location as compared to its sensitivity at other locations. For these reasons, the transient time at the channel center is selected as an indicator for the channel transient time. The transient time is estimated by marching in the time domain and searching for the time at which any further marching in time does not cause any significant change in the dimensionless velocity at the channel center. The change in this velocity is considered insignificant if the percentage change, defined as the difference between the new and the old velocities over the new one, is less than 1%.

Figs. 3 and 4 show samples for the transient spatial velocity distribution at different times and for two power law indices n. From these figures it is clear that the velocity profiles have a slug flattened form especially far from the wall. This is a typical behavior for the spatial velocity distribution in porous domains. This is due to the thin boundary layer encountered in porous domains. As a result, there are large gradients in the velocity

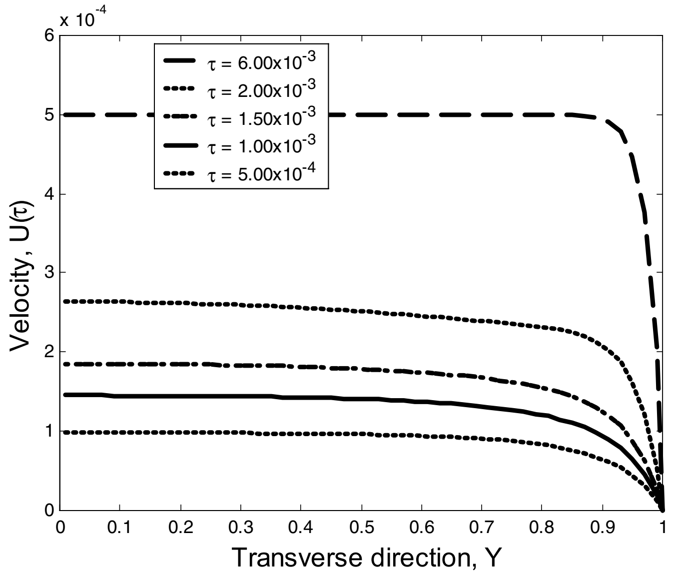

Fig. 3. Velocity spatial distribution at different times τ. $Da^* = 1 \times 10^{-2}$, $\Gamma = 10.0$, $n = 0.5$.

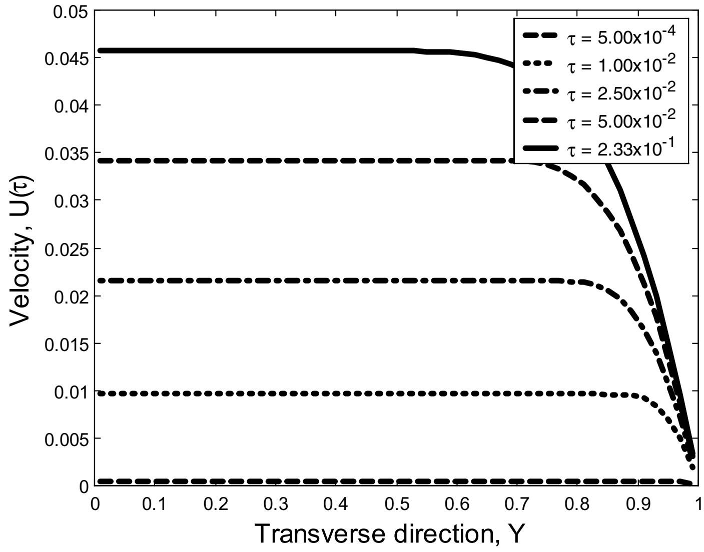

Fig. 4. Velocity spatial distribution at different times τ. $Da^* = 1 \times 10^{-2}$, $\Gamma = 10.0$, $n = 1.5$.

profile near the wall and this explains why early investigations adopted Darcy model which assumes that there is a slip, sudden change, in velocity near boundaries. It is clear from Figs. 3 and 4 that lower n has flatter profile than higher n. Also, it is clear from these two figures that fluids with higher n has higher velocity than that of lower n. This may be explained by writing Darcy version of Eq. (1) under steady state conditions. This Darcy version is obtained by dropping Brinkman and Forchheimer terms, and it is given as:

$$1 - \frac{1}{Da^*} U^n = 0$$

which is solved to yield:

$$U = (Da^*)^{1/n}$$

Now, with the notation that porous medium always has $Da^* < 1$, it is clear that fluids having higher n attain larger velocities than fluids having lower n.

Figs. 5–7 show the effect of Da^* on the transient time τ_t at different microscopic inertial numbers Γ and

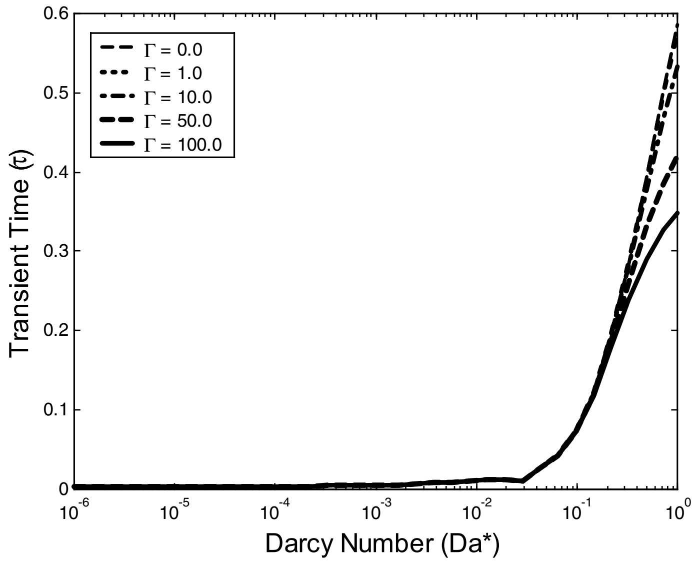

Fig. 5. Effect of Darcy number Da^* on the transient time τ_t at different Forchheimer numbers Γ. $n = 0.5$.

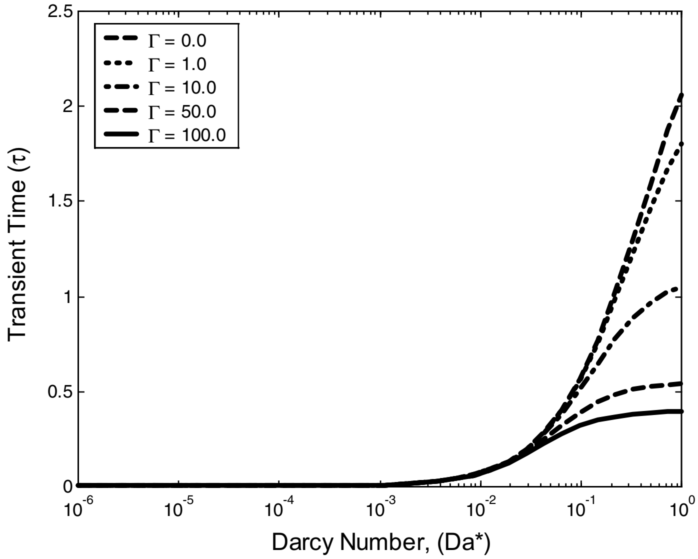

Fig. 6. Effect of Darcy number Da^* on the transient time τ_t at different Forchheimer numbers Γ. $n = 1.0$.

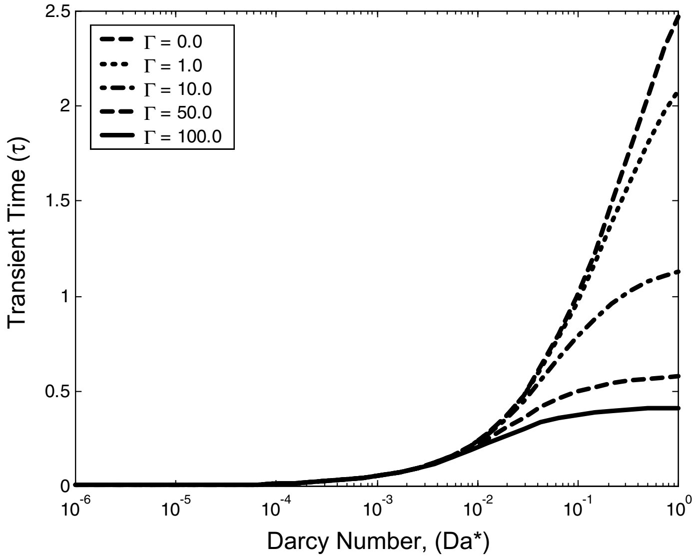

Fig. 7. Effect of Darcy number Da^* on the transient time τ_t at different Forchheimer numbers Γ. $n = 1.5$.

different power law indices n. It is clear that porous domains of low Darcy numbers ($<10^{-3}$) have very small transient times for all ranges of the microscopic inertial numbers Γ. This implies that the effect of the macroscopic local inertial term may be neglected in porous domains having $Da^* < 10^{-3}$. In the literature [7,8,11–13], it is found that the macroscopic local inertial term is of insignificant effects on the hydrodynamics behavior of Newtonian fluids flow in porous domains having relatively small values of Da numbers. Figs. 5–7 show that the macroscopic local inertial term has more significant effect on the channel hydrodynamics behavior in the case of non-Newtonian fluids having $n > 1$. The focus on this behavior will be revisited later again in Figs. 8 and 9. Channels having small Da^* numbers contain less amount of fluid due to its small void ratio ϵ. The permeability of the porous domain is proportional to its void ratio ϵ and as a result, the porous domain fluid content is proportional to Darcy number. As the mass of the fluid content in the porous domain decreases its local

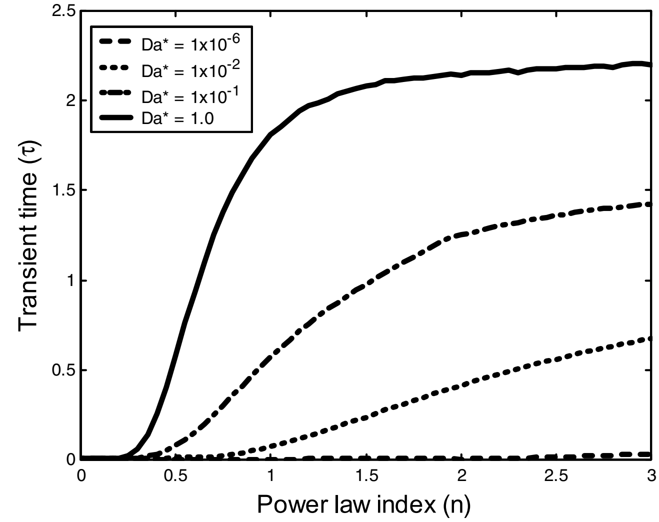

Fig. 8. Effect of power law index n on the transient time τ_t at different Darcy numbers Da^*. $\Gamma = 1.0$.

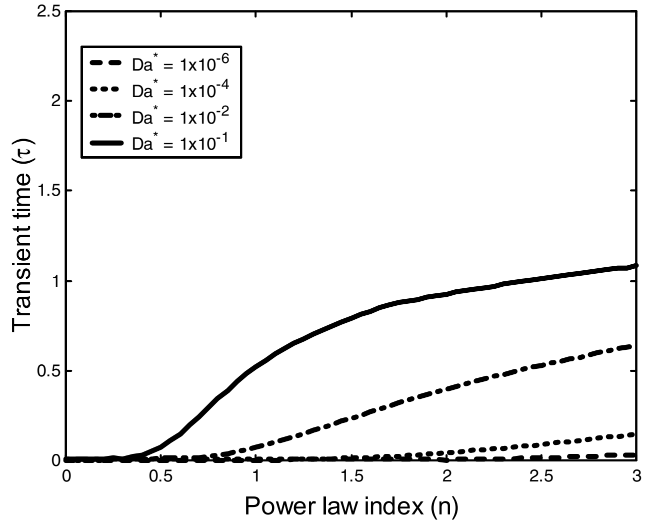

Fig. 9. Effect of power law index n on the transient time τ_t at different Darcy numbers Da^*. $\Gamma = 10.0$.

inertia decreases. As a result, shorter time is required by the fluid to attain the steady state behavior.

Figs. 5–7 show that the effect of the macroscopic local inertial term is more significant at small values of the microscopic inertial numbers Γ. As Γ increases, the effect of the macroscopic local inertial term becomes insignificant as compared to the effect of the microscopic Forchheimer inertial term. It is clear from Eq. (1) that the microscopic friction (Darcy) term, the macroscopic friction (Brinkman) term and the microscopic (Forchheimer) inertial term resist the pressure gradient driving force and as a result resist the acceleration in the fluid flow. As these three resistances increase, it will be much easier for the fluid to attain the steady state behavior because the steady velocity profile will have less values. As a result, shorter time is required by the channel to attain the steady hydrodynamics behavior. As an example, imagine the situation in which there is no resistance against the fluid flow. This reduces Eq. (1) to

$$\frac{\partial U}{\partial \tau} = 1 \tag{6}$$

which, with $U(0) = 0$, gives the following solution

$$U = \tau \tag{7}$$

This implies that the channel velocity increases and increases as time proceeds without any upper limit for this increase and hence, the transient time is of infinite value. The appearance of the microscopic (Forchheimer) inertial resistance ΓU^2, the microscopic frictional (Darcy) resistance $\frac{1}{Da^*}U^n$ and the macroscopic frictional (Brinkman) resistance $\frac{\partial}{\partial Y}[|\frac{\partial U}{\partial Y}|^{n-1}\frac{\partial U}{\partial Y}]$ shortens the time required by the channel to attain steady state behavior by reducing the upper limit of the steady velocity profile.

Also, it is clear from Figs. 5–7 that the effect of Da^* on the channel transient time is more significant at small values of Γ and large values of n. The effect of Γ on the transient time τ_t is more significant at large values of Da^* and n. Large values of Da^* and n implies that the porous domain contains more mass of thick (viscous) fluid, and the fluid transient time is more sensitive to the change in its microscopic inertial term.

Figs. 8 and 9 show the effect of the non-Newtonian power law index n on the channel transient time τ_t at different values of Da^* and Γ. The figures reveal that the macroscopic local inertial term has significant effect on the channel hydrodynamics behavior for relatively large values of n even when Da^* has relatively small values. As n increases, the fluid becomes thicker, i.e., more viscous. This implies that longer time is required by the fluid to attain the steady state behavior due to its slow response to the pressure gradient driving force. Also, it is clear from these figures that there is an upper limit for n beyond which changing n has insignificant effect on the channel transient time. This asymptotic behavior in τ_t with n becomes more clear at large values of Da^* and small values of Γ. The channel transient behavior is very sensitive to the change in n for $n < 1$. especially in channels having larger Da^*.

The same figures show that the macroscopic local inertial term has insignificant effect on the channel hydrodynamics behavior for all non-Newtonian fluids having $n < 0.5$ over the entire ranges of Da^* and Γ. This is also true for non-Newtonian fluids, having $n < 0.5$, flow through clear non-porous domains. This is concluded from the curves of Figs. 8 and 9 which have $Da^* > 0.1$. It is known in the porous domain literature that as Da^* increases, the porous domain approaches the clear non-porous one and porous domains having $Da^* > 0.1$ are considered anon-porous or clear ones [2,15]. Fluids having $n < 0.5$ are thin fluids having low viscosity and the response of these fluids to the imposed pressure gradient is very fast. As a result, the time required by these fluids to attain the steady behavior is very short.

5. Conclusion

Numerical solutions are obtained for the transient fluid flow problem in horizontal parallel-plate channels filled with porous medium under the effect of a suddenly imposed pressure gradient. The effect of the porous medium macroscopic local inertial term is investigated. It is found that the effect of the macroscopic local inertial term may be neglected in porous domains having $Da < 10^{-3}$ over the entire ranges of power law indices n and microscopic inertial numbers Γ. The macroscopic local inertial term has more significant effect on the channel hydrodynamics behavior in the case of non-Newtonian fluids having $n > 1$ and small values of Γ. Also, it is found that the effect of Da^* on the channel transient time is more significant at small values of Γ and large values of n. The effect of Γ on the transient time τ_t is more significant at large values of Da^* and n. It is noticed that there is an upper limit for n beyond which changing n has insignificant effect on the channel transient time. It is concluded that the macroscopic local inertial term has insignificant effect on the channel hydrodynamics behavior for all non-Newtonian fluids having $n < 0.5$ and over the entire ranges of Da^* and Γ.

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